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LETTER TO THE EDITOR

On the superfluid transition in dense electron systems

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Abstract. Kohn's singularity in dense electron systems is shown to give rise to instability against Cooper pair formation for large orbital momentum l, i.e. for $l \gg r_s^{-1/2}$ — while for dilute systems ($r_s \geqslant 1$) pairing with $l = 1$ is predicted.

It has long been known (Kohn and Luttinger 1965) that in dilute Fermi gas the use of an expansion for a bare vertex for a Cooper channel in powers of the gas parameter, which is entirely appropriate for slow particles in vacuum, becomes inappropriate when the fermionic background is taken into account: all the partial components Γ_l with $l \geqslant 1$ (l is the orbital momentum of the Cooper pair) prove to be of the same order of magnitude and, most importantly, to be negative for large l no matter what the sign of the force between the particles on the Fermi surface is. Hence the normal state of the Fermi liquid becomes unstable, at some finite temperature, against the formation of Cooper pairs. The physical reason for this effect is the screening of the initial interaction due to the fermionic background, which results in the appearance of a long-range oscillatory 'tail' even if one starts from purely δ-functional repulsion. In a recent Letter (Chubukov and Kagan 1988) we noted that the Kohn–Luttinger mechanism extends right up to $l = 1$ and, moreover, $|\Gamma_1|$ turns out to be maximal among all $|\Gamma_l|$ with $l \geqslant 1$; thus the dilute Fermi gas with a repulsive potential undergoes a transition to the superfluid state with $l = 1$. The critical temperature was found to be

$$T_1 \simeq \varepsilon_F \exp\{-[(5\pi^2/8)/(2 \ln 2 - 1)]/(ap_F)^2\} \tag{1}$$

($a > 0$ is the scattering length, $ap_F \ll 1$). If the scattering length is of the same order of magnitude as the diameter of the potential r_0 ($V = V_0 e^{-r/r_0}$)—that is, if the Born parameter $\gamma = mV_0 r_0^2/4\pi$ is of the order of unity—the corrections to the exponent will be small in powers of ap_F†, while for $\gamma \ll 1$, i.e. when $a \ll r_0$, we must additionally require that $ap_F \ll \gamma^2 \ll 1$ because only in this case does the contribution of the second-order diagrams to Γ_1 dominate over that of the p harmonics of the initial interaction.

The purpose of this Letter is to study the consequences of the Kohn–Luttinger effect in electron systems. The 'jelly' model in which the electron charge is compensated by uniformly spread positive charge will be used. A well known peculiarity of the Coulomb interaction is that the electron plasma is more nearly ideal when it is denser; this is why the theory is reliable only for high electron densities when $r_s = 1.92/p_F a_B \ll 1$ ($a_B = (me^2)^{-1}$ is the Bohr radius). For $r_s \ll 1$ we can use the Lindhard expression for the

† For the decreasing potentials $V \sim r^{-\alpha}$, equation (1) is correct for $\alpha > 4$ (see Chubukov and Kagan 1988).

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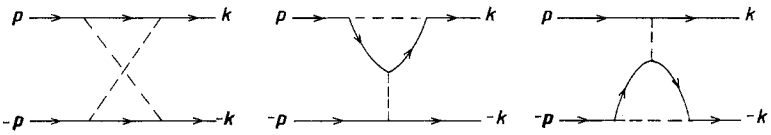

Figure 1. Exchange and crossing-type diagrams which give rise to the difference between $V_{\text{eff}}(\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{k})$ and the bare vertex for the Cooper channel.

dielectric function; this means that we deal only with fast particles, because the Thomas–Fermi momentum $\kappa = (6\pi ne^2/\varepsilon_F)^{1/2}$ is small in comparison with p_F when $r_s \ll 1$ ($\kappa^2/p_F^2 = 0.65 r_s \ll 1$). Finally, for $r_s \ll 1$ we need only consider the second-order diagrams because the Born parameter $\gamma = r_s^{1/2}/24 \ll 1$ (the role of V_0 is played by $e^2\kappa$).

The Kohn singularities arise from the zero-sound-type loop diagrams. Some parts of these diagrams are involved in the screeening of the Coulomb interaction: the singularity leads to non-analyticity in $\varepsilon(\boldsymbol{q}, \omega = 0)$ and, hence, in $V_{\text{eff}}(\boldsymbol{q})$ when the momentum transfer $\boldsymbol{q}$ is close to $2p_F$:

$$V_{\text{eff}}(\boldsymbol{q}) = 4\pi e^2/q^2\varepsilon(\boldsymbol{q}, \omega = 0) = 4\pi e^2 \Big/ \left[q^2 + \kappa^2 \left(\frac{1}{2} + \frac{p_F^2 - q^2/4}{2p_F q} \ln \frac{p_F + q/2}{p_F - q/2} \right) \right]. \tag{2}$$

The remaining zero-sound-type diagrams cause the difference between $V_{\text{eff}}(\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{k})$ and the bare vertex for the Cooper channel. These diagrams are presented as figure 1.

The broken lines represent $V_{\text{eff}}(\boldsymbol{q})$. All the diagrams contain singularities (non-analyticity in the expansion in powers of θ, the angle between the momenta of the incoming and outgoing particles) when θ is close to zero for the first diagram and close to $2p_F$ for the others. The fact that for $r_s \ll 1$ the particles are fast means that $V_{\text{eff}}(2p_F)/V_{\text{eff}}(0) \simeq r_s \ll 1$. This allows us to single out the first diagram in figure 1 and to neglect the other two as well as the non-analytical part of $V_{\text{eff}}(\boldsymbol{q})$. Kohn and Luttinger have shown that due to the singularity the partial components of the bare vertex for the Cooper channel Γ_l for large l drop off as described by a power law, l^{-4}, and not exponentially with l. The exchange type of the first diagram ensures attraction independently of the parity of l and for this reason the normal state of the Fermi liquid becomes unstable against pairing at $T = T_l$ where

$$T_l \sim \varepsilon_F \exp(-1/\lambda_l) \tag{3}$$

and

$$\lambda_l = (mp_F/4\pi)\Gamma_l = [(mp_F/4\pi)V_{\text{eff}}(0)]^2\, l^{-4}. \tag{4}$$

The calculation of T_l demands caution because, rigorously, $V_{\text{eff}} = V_{\text{eff}}(\boldsymbol{q}, \omega)$ and the frequency-dependent part of V_{eff} might be important. However, direct calculation shows that for large l the typical values of the intermediate momentum are of the order of p_F/l while typical frequencies are of the order of ε_F/l^2, so we can proceed to $q \to 0$, assuming $\omega/q \to 0$.

For Coulomb interaction $V_{\text{eff}}(0) = 4\pi e^2/\kappa^2$ and, accordingly, the quantity $\lambda_l = (\pi/4)^2 l^{-4}$ and does not contain any small parameter. Therefore, one can assume the transition temperature to be rather high, but unfortunately this is not the case. In fact, it is possible to single out the power-law contribution only for $l \gg r_s^{-1/2}$ and, as we have already mentioned, the typical values of q are of the order of p_F/l and going to the q-independent limit in (2) is possible only for $l \gg \kappa/p_F \sim r_s^{1/2}$. Also, the imposition of the

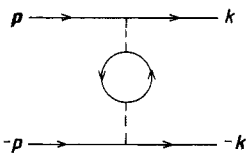

Figure 2. The diagram that gives rise to the singularity in V_{eff} in the limit $r_s \gg 1$.

Born approximation condition $\gamma \ll 1$ demands an additional increase of l in order to ensure the domination of the second-order-diagram contribution (leading to attraction) over the repulsive-type contribution from V_{eff}. Simple calculations show that the second-order contribution is dominant when

$$r_s(lr_s^{1/2})^{-4} > r_s(lr_s^{1/2})^{-1/2} \exp(-lr_s^{1/2}) \tag{5}$$

or when

$$l > l_c = r_s^{-1/2} |\ln r_s| [1 + \tfrac{7}{2}(\ln |\ln r_s|)/|\ln r_s| + \ldots] \tag{6}$$

which is to say the expression for λ_l for $r_s \ll 1$ of course indirectly contains small parameters.

With increasing r_s the critical value of l will decrease, thus leading to an increase of the transition temperature.

In real metals $2 \leqslant r_s \leqslant 6$. In this case, rigorous calculations cannot be carried out. Nevertheless, formally proceeding to the opposite limit, $r_s \gg 1$, in the expression (2) leads to the condition $V_{\text{eff}}(2p_F) = 2V_{\text{eff}}(0)$. This allows us to consider the situation opposite to the previous one—that is, to suppose as a very crude estimate that the bare vertex for the Cooper channel coincides with V_{eff}. The singularity in V_{eff} arises from the diagram representing the creation and annihilation of a virtual electron–hole pair (figure 2). The singular part of this diagram is proportional to $(-1)^{l+1}$ (Kohn and Luttinger 1965)—i.e. it promotes attraction for odd values of l. Numerical calculation shows that the contribution from the singular part of V_{eff} prevails over that from the non-singular part right up to $l = 1$, as in the case of weakly non-ideal Fermi gas, and the value of $|\Gamma_1|$ is maximal; hence, lowering the temperature leads to instability against p pairing. The critical temperature is estimated as

$$T_c = T_c^{(1)} \sim \varepsilon_F \exp(-15.1). \tag{7}$$

Note that, again, the parameter used in solving the problem does not appear in the final result because in this case we are dealing with slow particles and the scattering length that appears in the result in the combination ap_F is again connected with $V_{\text{eff}}(0)$; hence $ap_F = mp_F V_{\text{eff}}(0)/4\pi = \pi/4$. For typical values of the Fermi energy in metals $\varepsilon_F \sim 10^4$ K, $T_c^{(1)}$ proves to be 10^{-2} K. A more accurate theory would include the contributions from the other diagrams, of exchange and crossing types (see figure 1), as well as specification of an expression for V_{eff} (see, for instance, Singwi *et al* 1970).

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